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Effect of Suction on Unsteady MHD Free Convection and Mass Transfer Flow past a Continuous Permeable Sheet

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ABSTRACT

The effect of transpiration on the time-dependent MHD natural convective heat and mass transfer fluid flow past a continuous porous plate has been investigated numerically. We have changed the partial differential equations into ordinary differential equations by considering the similarity transformation. The changed governing ODEs were then solved numerically using the finite difference method through the shooting technique in ODE45 MATLAB software. The numerical results are shown graphically performing the impacts of several non-dimensional parameters/numbers entering into the current problem. The velocity of the fluid diminishes for improving values of the inclined angle, magnetic force parameter, suction parameter, Schmidt number, and Prandtl number but the inverse situation is found in the velocity profile for enhancing values of the Darcy number. The temperature of the fluid lessens for improving values of the Prandtl number and the suction parameter. With rising values of the suction parameter and Schmidt number the fluid concentration diminishes. With increasing values of the suction parameter from 0.5 to 1.0 and the Prandtl number from 0.71 to 1.0, the heat transfer rate improves by about 36% and 25%, respectively. The local skin friction coefficient enhances by about 32% for augmented values of the Darcy number from 0.5 to 2.0. On the other hand, with enhancing values of the suction from 0.5 to 3.0, the inclined angle from 0^0 to 60^0, Prandtl number from 0.71 to 7.0, and Schmidt number from 0.22 to 0.67 the local skin friction coefficient diminishes about 40%, 67%, 33% and 17% respectively. The mass transfer rate enhances about 68% and 90% due to improving values of the suction parameter from 0.5 to 3.0 and Schmidt number from 0.22 to 0.67, respectively.

Keywords: MHD, Continuous sheet, Suction, Heat and mass transport, Permeability

1. Introduction

Magnetohydrodynamic (MHD) fluid flow past a continuous permeable sheet is a part and parcel of engineering, pure science, biomedical fields and technology for example MHD accelerators, MHD power generators, ionized gases, electrolytes, metal-working processes, traveling wave tubes, control fusion research, blood flow measurements, and propulsion units. A continuous sheet is a plate that is rotated through different angles concerning time. A medium or sheet having a porous surface is treated as a permeable sheet. In nature, we have rocks, soil, wood, cork, etc. which are examples of porous media. Petroleum technology, rock abd soil mechanics, material science, human lungs, the dying process, and small blood capillaries consist of the permeable medium. A continuous permeable sheet not only has the property named porosity but also indicates permeability. The amount of fluid getting to be gripped by the material is the key concern of porosity, while the quantitative potentiality of the permeable medium to allow fluid flow is evaluated by the permeability. Ali et al. [1] solved the governing equations for a time-dependent MHD natural viscous convective heat abd mass transfer fluid flow along a continuous surface. Alam [2] studied the Dufour and Soret effects on steady two-dimensional hydromagnetic natural convective heat and mass transfer flow in an inclined stretching sheet. Char [3] analyzed the transpiration effect on steady laminar boundary layer and heat transfer flow past a continuous stretching surface. The effects of thermal radiation, heat source, radiation absorption, and mass diffusion on the time-dependent magnetohydrodynamic free convective micropolar fluid flow with nanoparticles past a vertical porous sheet has been examined by Arifuzzaman et al. [4]. Quite recently Hasanuzzaman et.al [5] investigated the heat-mass transport through a vertical permeable plate for unsteady magneto-convective with the Dufour and thermal diffusion effects. Hasanuzzaman et al. [6] extended Hasanuzzaman et.al [5] by considering the additional term thermal radiation parameter. In this paper, we extended Hasanuzzaman et.al [5] research by considering the inclined porous plate.

The purpose of this paper is to estimate the past problems of an inclined permeable plate taking into account mass transfer including MHD free convection and suction effects. The velocity, temperature, and concentration profiles are described carefully for various values of the governing numbers/parameters graphically. Also, the effects of the governing parameters/ numbers on the local skin friction coefficient, mass transfer rate and heat transfer rate are presented in tabular forms.

2. Governing Equations

Let us consider the time-dependent 2D hydromagnetic natural of an electrically conducting, incompressible viscous fluid flow past an inclined porous plate with an acute angle γ to the vertical as shown in Fig. 1. The fluid flow is taken to be in the x-

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axis. The x-axis is considered along the inclined porous plate. This inclined porous plate is perpendicular to the y-axis. The direction of the fluid flow is considered normal to a uniform magnetic field of strength B_0. The fluid is considered to have fixed properties except that the effect of the density variations with concentration and temperature, which are assumed only in the body force term. Under the above suppositions, y and t are the functions of the physical variables only. In this numerical simulation, we consider the Reynolds number related to the magnetic field is much smaller than unity so that the imposed magnetic field is insignificant compared to the imposed magnetic field (Pai, [7]).

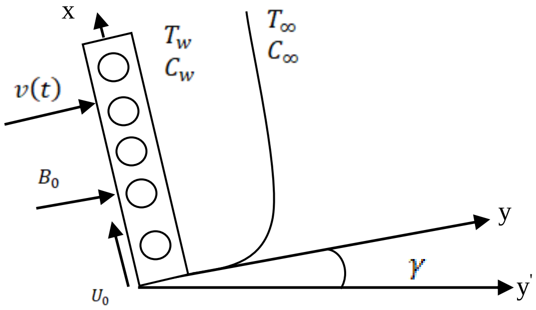


Fig.1: Physical model and coordinate systems

Using the above assumptions and applying the Boussinesq approximation, the governing equations are given by (Hasanuzzaman et.al [5]):

The Continuity Equation:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

The Momentum Equation:

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta\cos\gamma(T - T_\infty) + g\beta^*\cos\gamma(C - C_\infty) - \frac{\sigma' B_0^2 u}{\rho} \tag{2}$$

The Energy Equation:

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{k_T D_m}{C_p C_c}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

The Concentration Equation:

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{k_T D_m}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

subject to the boundary conditions

$$t > 0, u = U_0(t), T = T_w, v = v(t), C = C_w \text{ at } y = 0 \tag{5}$$

$$t > 0, u = 0, T \to T_\infty, v = 0, C \to C_\infty \text{ at } y \to \infty \tag{6}$$

where γ is the inclined angle, T is the fluid temperature, C is the concentration of the fluid, ρ is the density of the fluid, υ is the fluid kinematic viscosity, g is the acceleration due to gravity, β is the thermal expansion coefficient due to temperature, β^* is the concentration expansion coefficient, T_w is the temperature of the wall, C_w is the concentration of the wall, T_∞ and C_∞ are the temperature and concentration in the free stream, respectively, k is the thermal conductivity of the porous plate, C_p is the specific heat at constant pressure, D_m is the mass diffusivity coefficient, k_T is the thermal diffusion ratio, T_m is the mean temperature of the fluid, and the velocity components along the x and y direction are u and v, respectively.

A similarity parameter σ is introduced by

$$\sigma = \sigma(t) \tag{7}$$

Where σ is unsteady length scale. The solution of the continuity equation (1) is considered in terms of this length scale given by:

$$v = -v_0\frac{\upsilon}{\sigma} \tag{8}$$

Here v_0 is the non-dimensional normal velocity at the porous plate. $v_0 < 0$ indicates blowing and $v_0 > 0$ indicates suction.

The similarity transformations are given by:

$$\eta = \frac{y}{\sigma}, f(\eta) = \frac{u}{U_0}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{9}$$

Using the equations (7)-(9), the equations (1)-(4) are changed into the non-dimensional coupled ODEs as follows:

$$f'' + 2\xi f' + Gr\cos\gamma\theta + Gm\cos\gamma\phi - Mf = 0 \tag{10}$$

$$\theta' + \Pr(2\xi\theta' + Df\phi'') = 0 \tag{11}$$

$$\phi'' + 2\xi S_c\phi' + S_c S_0\theta'' = 0 \tag{12}$$

The transformed boundary conditions are given by:

$$f = 1, \theta = 1, \phi = 1 \text{ at } \eta = 0 \tag{13}$$

$$f = 0, \theta = 0, \phi = 0 \text{ at } \eta \to \infty \tag{14}$$

where $G_r = \frac{g\beta(T_w - T_\infty)\sigma^2}{U_0 v}$ is the Grashof number, $M = \frac{\sigma' B_o{}^2\sigma^2}{\rho v}$ is the Magnetic force parameter, $G_m = \frac{g\beta^*(C_w - C_\infty)\sigma^2}{U_o\upsilon}$ is the modified Grashof number, $\Pr = \frac{\rho\upsilon C_p}{k}$ is the Prandtl number, $D_f = \frac{D_m k_T(C_w - C_\infty)}{C_s C_p \upsilon(T_w - T_\infty)}$ is the Dufour number, $S_0 = \frac{D_m k_T(T_w - T_\infty)}{\upsilon T_m(C_w - C_\infty)}$ is the Soret number, $S_c = \frac{\upsilon}{D_m}$ is the Schmidt number and $\xi = \eta + \frac{v_0}{2}$ is the time-dependent parameter.

The local skin friction coefficient f', the Nusselt number Nu, and the Sherwood number Sh are given by:

$$\tau \propto f', \;\; Sh \propto -\phi', \;\; Nu \propto -\theta' \tag{15}$$

3. Results and Discussions

The effect of suction on the time-dependent 2D free MHD convective mass and heat transfer fluid flow past an inclined porous sheet are investigated numerically. The non-dimensional temperature, velocity, and concentration profiles are obtained from equations (10)-(12). This temperature, velocity, and concentration distributions are displayed in Figs. 2-11 for seperate values of the inclined angle (γ),the magnetic force parameter (M), the Darcy number (Da), the Schmidt number (Sc), the suction parameter (v_0), and the Prandtl number (Pr). To be practical, the values of Schmidt number Sc are taken for water-vapour (Sc = 0.60) and hydrogen (Sc = 0.22). The values of Prandtl number Pr are chosen for electrolyte solution for example water (Pr=7.0), air (Pr=0.71) , and salt water (Pr= 1.0). Finally, the values of the dimensionless parameters/numbers are chosen arbitrarily.

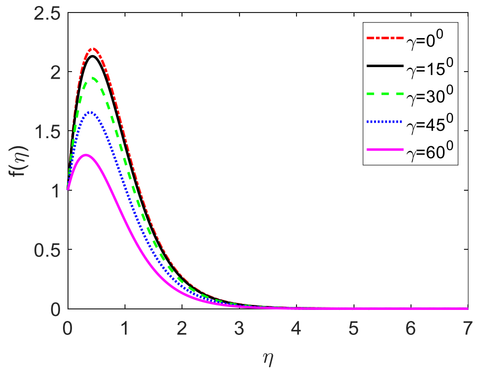


Figure 2: Velocity profile for γ

The influence of several inclinations (γ) of the porous plate on the velocity distribution is depicted in Fig.2. It is found from Fig.2 that the velocity of the fluid is improved for improving values of the inclined angle. When the porous plate is vertical ($\gamma = 0^0$), then the fluid flow has a maximum velocity. But, when the porous plate is started to move, then the fluid velocity reduces. The coefficient of local skin friction lessens by about 67% due to rising values of the inclined angle from 0^0 to 60^0. Physically, the buoyancy force reduces the gravity components, as the sheet is inclined.

Fig.3 illustrates the velocity profile for seperate values of the suction parameter (v_0) on. The effect of the suction parameter in the computational domain means some fluid suck from this domain. Hence, the fluid can't move freely in the computation domain because the viscous force augmentes for uplifting values of v_0. The local skin friction coefficient lessens by about 21% due to improving values of v_0 from 0.5 to 2.0 Finally, the velocity of the fluid lessens for improving values of the suction parameter. The suction presents the usual fact that the suction stabilizes the growth of the boundary layer.

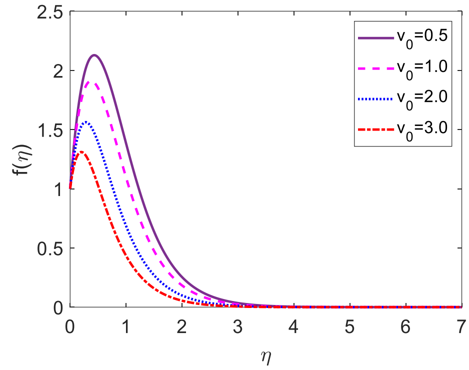


Figure 3: Velocity profile for v_0

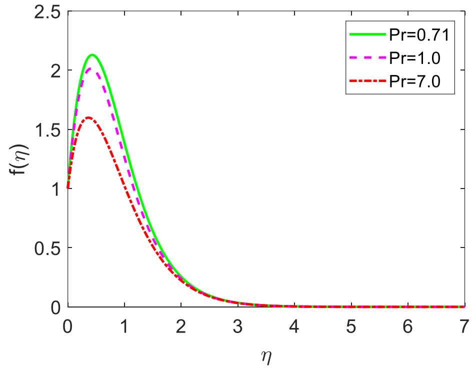
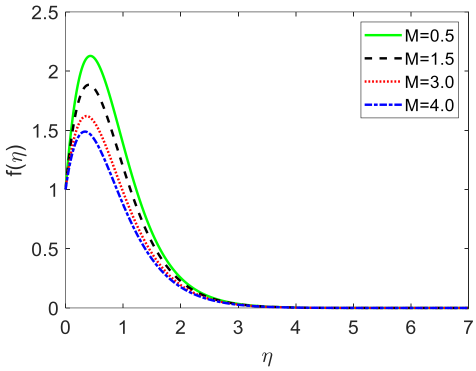


Figure 4: Velocity profile for Pr

Fig.4 shows the velocity profile for several values of the Prandtl number (Pr). The ratio of the kinematic viscosity and the thermal conductivity is called the Prandtl number. The mathematical formula of the Prandtl number is $\mathrm{Pr} = \dfrac{\rho \upsilon c_p}{k}$. The kinematic viscosity is proportional to the Prandtl number. It is concluded from Fig.4 that with an increasing vlues of the Prandtl number the kinematic viscosity of the fluid improves. When the fluid kinematic viscosity improves the frictional force exists in the computation domain. The skin friction coefficient reduces by about 4% for rising values of the Prandtl number from 0.71 to 1.0. The fluid can't move freely for improving the frictional force. Hence the fluid velocity lessens for improving the values of the Prandtl number. Due to enhancing the Prandtl number, the fluid kinematic viscosity improves which in turn creates the fluid much thicker.

Fi. 5 depicts the influence of several values of the magnetic force parameter (M) on the velocity distribution. A drug force creates an electrically conducting fluid for improving values of the magnetic force parameter. This drug force is called the Lorentz force. This force works opposite the fluid flow when the magnetic force is imposed in the normal direction.

Hence, the fluid diminishes for rising values of the magnetic force parameter in Fig.5.



Figu.5: Velocity profile for M

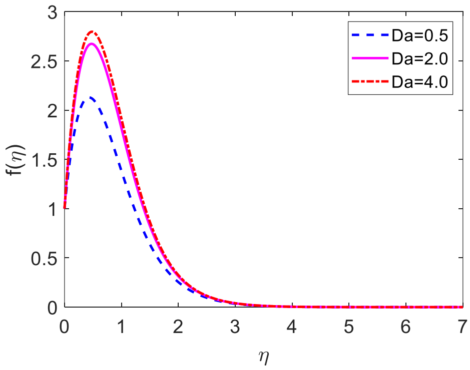


Fig.6: Velocity profile for Da

Fig.6 depictes the velocity profile for different values of Darcy number (*Da*). It is noticed from Fig.6 that the fluid velocity lessens with an enhancement of *Da*. The porosity of the porous medium enhances for huge values of *Da*. For this reason, the fluid flows improve quickly.

The ratio of the kinematic viscosity and the coefficient of the mass diffusivity is said to be the Schmidt number. The mathematical formula of the Schmidt number is defined by $Sc = \frac{\upsilon}{D_m}$. Figure 7 presents the velocity profile for seperate values of Sc. The Schmidt number is proportionate to the kinematic viscosity. The kinematic viscosity improves for uplifting values of the Schmidt number augmentes. It means the frictional force improves in the computational domain. The local skin friction coefficient reduces by about 6% due to rising the values of the Schmidt number from 0.22 to 0.33. So, the fluid flow has lower velocity for rising values of the Schmidt number as shown in Figure 7.

The impact of the Prandtl number ($\Pr$) on the temperature profile is displayed in Fig.8. The thermal conductivity is inversely proportionate to the Prandtl number. The thermal conductivity reduces for increasing values of the Prandtl number. The heat transfer rate improves for rising values of $\Pr$. The heat transfer rate increases by about 25% for moving values of the Prandtl number from 0.71 to 1.0. So, the fluid temperature decreases for rising values of the Prandtl number as shown in Figure 8. Physically, the fluids at a high value of the Pr have weak thermal diffusivity results a diminishing of the temperature of flowing fluid.

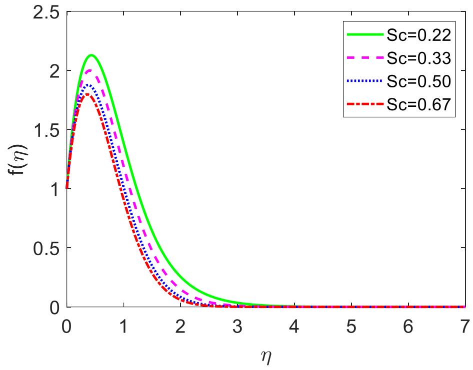


Fig.7: Velocity profile for Sc

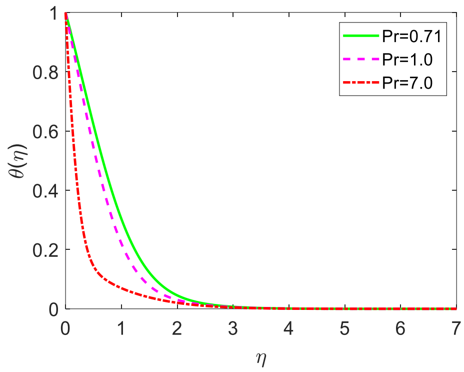


Figure 8: Temperature profile for Pr

The effect of the several values of the suction parameter (v_0) on the temperature distribution is displayed in Figure 9. From Fig.9, it is observed that the temperature of the fluid diminishes for improving the values of v_0. The heat transfer rate augmentes by about 55% for rising values of v_0 from 0.5 to 2.0. The amount of fluid reduces in the computational domain which means the heat transfer rate improves for rising values of suction.

The effect of separate values of the Schmidt number (Sc) on the concentration profile is illustrated in Fig.10. The Schmidt number is inversely proportionate to the mass diffusivity. When the Schmidt number improves then the fluid mass quickly reduces and also reduction of the concentration boundary layer thickness. It means the mass transfer rate improves for improving values of the Schmidt number. The mass transfer rate enhances by about 25% for rising values of Sc from 0.22 to 0.33.

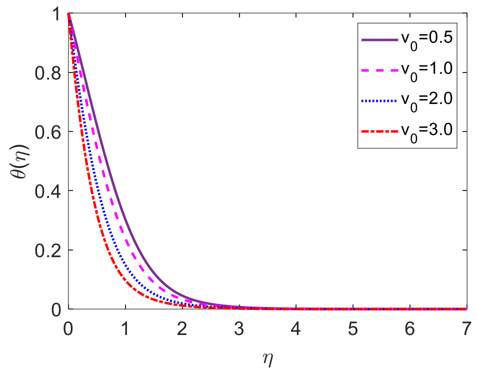


Figure 9: Temperature profile for v_0

Figure 11 shows the concentration profile for various values of the suction parameter v_0 . From Figure 11, it can conclude that with increasing values of suction the fluid concentration diminishes. The amount of fluid in the computational domain reduces which means the mass transfer rate also improves for rising values of suction. The mass transfer rate improves by about 39% for moving values of v_0 from 0.5 to 2.0. Sucking slows down fluid particles past the porous sheet and lessens the concentration of boundary layers' growth.

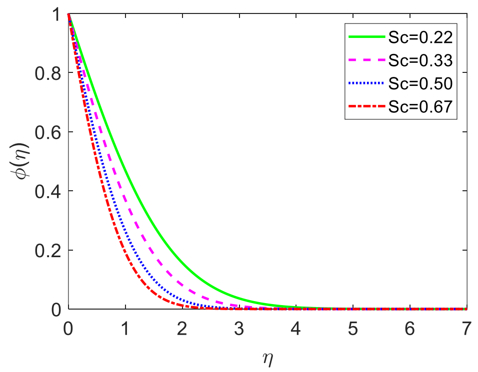


Figure 10: Temperature profile for Sc

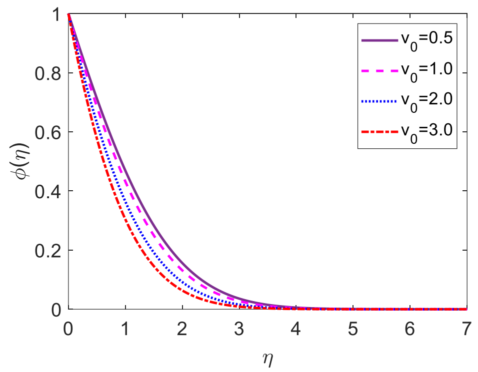


Figure 11: Concentration profile for v_0

Table 1: Numerical values of $f'(0)$, $-\theta'(0)$ and $-\varphi'(0)$ for different values of the inclined angle (γ)

γ	$f'(0)$	$-\theta'(0)$	$-\varphi'(0)$
0^0	6.364	0.706	0.601
15^0	6.072	0.706	0.601
30^0	5.217	0.706	0.601
45^0	3.856	0.706	0.601
60^0	2.082	0.706	0.601

Table 2: Numerical values of $f'(0)$, $-\theta'(0)$ and $-\varphi'(0)$ for different values of the suction parameter

v_0	$f'(0)$	$-\theta'(0)$	$-\varphi'(0)$
0.5	6.072	0.706	0.601
1.0	5.727	0.962	0.677
2.0	4.811	1.509	0.836
3.0	3.672	2.086	1.009

Table 3: Numerical values of $f'(0)$, $-\theta'(0)$ and $-\varphi'(0)$ for several values of the Prandtl number

Pr	$f'(0)$	$-\theta'(0)$	$-\varphi'(0)$
0.71	6.072	0.706	0.601
1.0	5.726	0.878	0.601
7.0	4.095	3.603	0.601

Table 4: Numerical values of $f'(0)$, $-\theta'(0)$ and $-\varphi'(0)$ for separate values of the Schmidt number

Sc	$f'(0)$	$-\theta'(0)$	$-\varphi'(0)$
0.22	6.072	0.706	0.601
0.33	5.704	0.706	0.757
0.5	5.329	0.706	0.964
0.67	5.071	0.706	1.147

The skin friction coefficient, heat and mass transfer rates for separate values of the non-dimensional parameters/numbers are displayed in Tables 1 to 4. It is concluded from tables 1 to 4 that with an increase values of the inclined angle, suction parameter, Schmidt number and Prandtl number the local skin friction lessens. The mass transfer rate rises for uplifting values of the Schmidt number, and suction parameter. The heat transfer rate improves due to incrasing values of the Prandtl number and the suction parameter.

5. Conclusions

The time-dependent MHD free convection and mass transfer flow past an inclined permeable sheet has been performed under the suction parameter effect. From the above study, it can concluded that:

- The velocity slows down for uplifting values of the inclined angle.
- Rising values of the suction parameter the velocity, temperature, and concentration of the fluid reduce.
- The skin friction coefficient decreases about 40%, 67%, 33%, and 17% for enhancing values of the suction parameter from 0.5 to 3.0, incline angle from 0^0 to 60^0, Prandtl number 0.71-7.0) and Schmidt number from 0.22 to 0.67, respectively.
- The heat transfer rate improves about 36% and 25% for improving values of the suction parameter from 0.5 to 1.0 and Prandtl number from 0.71 to 1.0, respectively.
- Increasing values of the suction parameter from 0.5 to 3.0 and Schmidt number from 0.22 to 0.67 the mass transfer rate enhances about 68% and 90%, respectively.

The results of this present paper can be useful for petroleum technology, soil and rock mechanics, material science, human lungs, blood flow measurements, electrolytes, control fusion research, the dying process, traveling wave tubes, metal-working processes, ionized gases, and propulsion units, etc.

6. References

1. Ali, F., Khan, I., & Shafie, S. (2013). Conjugate effects of heat and mass transfer on MHD free convection flow over an inclined plate embedded in a porous medium. *Plos one*, *8*(6), e65223.
2. Alam, M. S. (2016). Effect of Thermophoresis on MHD free convective heat and mass transfer flow along an inclined stretching sheet under the influence of Dufour-Soret effects with variable wall temperature. *Science & Technology Asia*, 46-58.
3. Char, M. I. (1988). Heat transfer of a continuous, stretching surface with suction or blowing. *Journal of Mathematical Analysis and Applications*, *135*(2), 568-580.
4. Arifuzzaman, S. M., Mehedi, M. F. U., Al-Mamun, A., Biswas, P., Islam, M. K., & Khan, M. (2018). Magnetohydrodynamic micropolar fluid flow in presence of nanoparticles through porous plate: A numerical study. *International Journal of Heat and Technology*, *36*(3), 936-948.
5. Hasanuzzaman, M., Azad, M., Kalam, A., & Hossain, M. (2021). Effects of Dufour and thermal diffusion on unsteady MHD free convection and mass transfer flow through an infinite vertical permeable sheet. *SN Applied Sciences*, *3*(12), 1-11.
6. Hasanuzzaman, M., Ahamed, T., & Miyara, A. (2022). Thermal Radiation Effect on Unsteady Magneto-Convective Heat-Mass Transport Passing in a Vertical Permeable Sheet with Chemical Reaction. *Computational and Mathematical Methods in Medicine*, *2022*.
7. Pai, S. I. (1962). Magnetogasdynamics and Electromagnetogasdynamics. In *Magnetogasdynamics and Plasma Dynamics* (pp. 27-40). Springer, Vienna.

Nomenclature

u	velocity component along the x-axis, ms^{-1}
v	velocity component along the y-axis, ms^{-1}
MHD	hydromagnetic
B	uniform magnetic field, Am^{-1}
J	density of current
T	temperature of fluid, k^{-1}
T_w	wall temperature, k^{-1}
T_∞	free stream temperature
C	fluid concentration, kg m^{-3}
C_w	wall concentration, kg m^{-3}
C_∞	free stream concentration
ρ	fluid density, kg m^{-3}
$U_0(t)$	uniform surface velocity
$v(t)$	suction velocity
g	acceleration due to gravity, ms^{-2}
υ	kinematic viscosity, m^{-2} s^{-1}
k	thermal conductivity
C_p	specific heat at constant pressure
k_T	thermal diffusion ratio
D_m	mass diffusivity coefficient
σ	similarity parameter
v_0	suction and blowing
Gm	modified Grashof number
Gr	Grashof number
M	Magnetic parameter
Pr	Prandtl number
S_0	Soret number
D_f	Dafour number
S_c	Schmidt number
t	Time, s
τ	skin friction coefficient
N_u	Nusselt number
S_h	Sherwood number
f	dimensionless velocity
θ	dimensionless temperature
ϕ	dimensionless concentration
γ	Inclined angle
ξ	time dependent parameter